In conformity with the existence theorem for implicit functions, the necessary condition for solvability of the system (5.6) for h^i and τ is

$$\partial(\Phi^1, \Phi^2, \Phi^3, \Phi^0)/\partial(h^1, h^2, h^3, \tau) \neq 0, \tag{5.7}$$

where Φ^0, Φ^i are sufficiently smooth functions of their arguments.

Evaluating the derivatives in (5.7), and utilizing the relationship

$$\frac{\partial \Phi^0}{\partial h^j} + \frac{1}{2} h_i \frac{\partial \Phi^i}{\partial h^j} = 0,$$

the inequality (5.7) can be written in the form

$$\Theta \det \left\| (\rho G)^2 g_{ij} - \left(\rho F^a_m n_a\right) \frac{\partial^2 U}{\partial F^i_m \partial F^j_n} \left(\rho F^b_n n_b\right) \right\| \neq 0. \tag{5.8}$$

Comparing (5.8) and (4.4), we find that the desired condition is $\rho G \neq \rho c$, i.e., the shock velocity should not equal the propagation velocity of the characteristic surface.

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TAKING INTO ACCOUNT THE STRUCTURAL INHOMOGENEITY OF A COMPOSITE MATERIAL IN ESTIMATING ADHESIVE STRENGTH

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One of the basic characteristics of a composite material is its adhesive strength. The experimental determination of this characteristic (in the case of a fiber composite) can be based on a measurement of the load, for which a fiber is pulled out of the matrix.

In order to correctly calculate the adhesive strength from the results of such tests, it is necessary, however, to solve a complex mechanical problem of the distribution of contact stresses between the fiber and the matrix. The use of rigorous methods for analyzing composites at the constituent component level does not permit obtaining at the present time an exact analytic solution of the corresponding problem in the theory of elasticity. For this reason, the engineering approach [1-3], in which it is assumed that the fibers function only under tension, while the matrix only functions under shear, is widely used. Evidently, with this method, it is impossible to take into account the possible singularity of the contact stresses at locations where the fiber and the matrix join on the free boundary. In addition, in using a simplified model, there arises the natural problem of the limits of applicability of the corresponding solutions even outside the regions of concentrated stresses.

A detailed representation of the stressed state can be obtained by the finite-element method. However, in order to apply numerical methods efficiently, preliminary analytic solutions, which correctly reflect the basic

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characteristics of the distribution of stresses in the composite material, are also important. This paper is concerned with constructing such analytic solutions taking into account the structural inhomogeneities of the material. First, we estimate the limits of applicability of the discrete model of the composite mentioned above and representations are obtained for the contact stresses, valid everywhere, except in the vicinity of singular points. These representations, taken together with the singular solutions constructed at the second stage, give an approximate solution to the problem that is equally useful in the entire contact region.

1. In order to establish the limits of applicability of the discrete model or a composite material, we shall examine the two-dimensional problem of pulling a fiber out of a matrix, shaped like a halfstrip ($0 \le x < \infty$, $-b \le y \le b$), whose finite boundaries ($y = \pm b$) are clamped. The fiber, treated as a one-dimensional elastic rod, is situated at the center of the half-strip and coincides with the Ox axis. A scheme describing line contact is used. The matrix, generally speaking, is assumed to be orthotropic and the principle directions of elasticity coincide with the Cartesian x and y axes.

In this formulation, the problem reduces to integrating the equations of equilibrium of the matrix

$$B_1 u_{xx} + G u_{yy} + (\nu_2 B_1 + G) v_{xy} = 0,$$
$$G v_{xx} + B_2 v_{yy} + (\nu_1 B_2 + G) u_{xy} = 0$$

with the following boundary conditions:

$$\sigma_{11} = \sigma_{12} = 0 \ (x = 0),\ u = U,\ v = 0 \ (y = 0),\ u = v = 0 \ (y = \pm b).$$

At infinity, all functions vanish. Here u and v are the components of the displacement vector of the matrix; $B_1 B_2$, tension and compression moduli of the matrix; G, shear modulus; σ_{11}, σ_{12}, axial and tangential stresses in the matrix; ν_1, ν_2, Poisson coefficients; and the indices x and y indicate differentiation with respect to the corresponding coordinates.

The displacements U(x) of the points of the fiber satisfy the equation

$$EFU_{xx} = P_0 \delta(x) - 2\tau(x), \tag{1.1}$$

where EF is the tensile strength of the fiber; $\delta(x)$, Dirac function; P_0, force applied to the fiber at the boundary point $x = 0$; $\tau(x)$, contact stress between the fiber and the matrix. Since $v = 0$ ($v_x = 0$) at $y = 0$,

$$\tau(x) = G u_y |_{y=0}. \tag{1.2}$$

In order to study the problem formulated above, which cannot be solved exactly, we shall apply the asymptotic method [4-6]. Since the contact stress (1.2) is defined only in terms of the function u, in accordance with the separation of the stress–strain state of the matrix [4-6], the solution (in the first approximation) reduces to integrating Eq. (1.1), taking into account (1.2) and the approximate equation of equilibrium of the matrix

$$\omega^2 u_{xx} + u_{yy} = 0 \ (\omega^2 = B_1/G), \tag{1.3}$$

to which the following boundary conditions correspond:

$$u_x = 0 \ (x = 0),\ G u_y = \tau(x) \ (y = 0),\ u = 0 \ (y = \pm b). \tag{1.4}$$

At infinity, all functions vanish.

Applying the cosine Fourier transformation with respect to the coordinate x to Eq. (1.3), solving the ordinary differential equation obtained taking into account the transformed equalities (1.1), (1.2), and (1.4), and finding the inverse transformation, we obtain

$$u(x, y) = -\frac{2P_0}{\pi EF} \int_0^\infty \frac{\operatorname{ch}(\omega y s) - \operatorname{cth}(\omega b s)\operatorname{sh}(\omega y s)}{s\,[s + g \operatorname{cth}(\omega b s)]} \cos xs\, ds,$$
$$\tau(x) = \frac{P_0 g}{\pi} \int_0^\infty \frac{\cos xs}{s \operatorname{th}(\omega b s) + g}\, ds,\ g = \frac{2G\omega}{EF}. \tag{1.5}$$

For small s, which corresponds to large values of the coordinate x, $\tanh(\omega bs) \approx \omega bs$, while the stress $\tau(x)$ takes the form

$$\tau(x) = \frac{P_0 g_*^2}{\pi} \int_0^\infty \frac{\cos xs}{s^2 + g_*^2}\, ds = \frac{P_0 g_*}{2} e^{-g_* x},\ g_*^2 = \frac{g}{\omega b} = \frac{2G}{EFb}. \tag{1.6}$$

The contact stress (1.6) corresponds to the solution of the problem assuming that the matrix functions only under shear.

For large s, which corresponds to small values of the coordinate x, $\tanh(\omega bs) \approx 1$, while the stress $\tau(x)$ from (1.5) is written in the form

$$\tau(x) = \frac{P_0 g}{\pi}\int_0^\infty \frac{\cos xs}{s+g}\,ds = -\frac{P_0 g}{\pi}(\cos gx \operatorname{ci} gx + \sin gx \operatorname{si} gx), \tag{1.7}$$

where si and ci are the sine and cosine integrals. The distribution of contact stresses, defined by Eq. (1.7), corresponds to the solution of the problem of pulling a fiber out of a semiinfinite matrix [5] and, in addition, $\tau(x)$ has, evidently, a logarithmic singularity at the point $x = 0$.

Thus, the solution obtained based on the simplified analysis, when it is assumed that the fibers function only under tension, while the matrix functions only under shear, is valid only for sufficiently large values of the coordinate x.

Analogous results can be obtained by examining the problem of pulling a fiber, situated along the x axis, out of a rectangular matrix ($0 \le x \le h$, $-b \le y \le b$) fixed along the edges $y = \pm b$, when $\sigma_{11} = \sigma_{12} = 0$ at $x = 0$ and h. In particular, the contact stress between the fiber and the matrix is defined by the equation

$$\tau(x) = \frac{P_0 g}{\pi}\sum_{n=1}^{\infty}\frac{\cos(n\pi x h^{-1})}{n \operatorname{th}(\omega n\pi b h^{-1}) + gh\pi^{-1}}. \tag{1.8}$$

If the matrix functions only under shear, then

$$\tau(x) = \frac{P_0 g_*}{2}\frac{\operatorname{ch}[g_*(h-x)]}{\operatorname{sh}(g_* h)}. \tag{1.9}$$

For $h \to \infty$, the solutions (1.8) and (1.9) go over into (1.5) and (1.6).

In the preceding examples, it was assumed that in a fibrous composite material, the fibers that are adjacent to the ones that are pulled out are fixed. The effect of these fibers, as shown above, is important only for sufficiently large values of the coordinate x, where the solution obtained based on the simplified analysis is valid; for this reason, the real structure of the composite can be taken into account quite simply. The corresponding calculations show that the approximation of fixed neighboring fibers is completely acceptable.

The region of values of the parameter x (and coordinate x), in which the discrete model of the composite is applicable, is estimated as follows: $s \lessdot g_*$ $(x \gtrdot g_*^{-1})$.

The discrete model of the composite material is not applicable in the zone next to the edge of the strip $(x < g_*^{-1})$. Here, it is necessary to use solutions (1.5) and (1.8), obtained by an asymptotic method and valid everywhere, except in the direct vicinity of the fiber–matrix joint at the free boundary. In this vicinity, a special solution must be constructed based on the exact equations of the theory of elasticity. If the rod model of a fiber is retained in this case, then we arrive at a correction to the logarithmic singularity of the contact stresses due to the power-law factor, which depends on the Poisson coefficients ν_1, ν_2 [7]. This factor equals 1 only for $\nu_1 = \nu_2 = 0$ and solutions (1.5) and (1.8) are valid in the entire contact region. If, on the other hand, in the region examined, the stressed state of the fibers can be described by the equations of the two-dimensional stressed state, then the singularity of the contact stresses (which does not depend on the transverse dimension of the fiber) is determined by the method of transferring the load to the fiber (stresses or displacements are given). When the stresses are given, the characteristic value λ, determining the exponent of the singularity, of the contact stresses $[\tau(x) = Ax^\lambda]$ in the vicinity of the fiber–matrix joint at a free boundary, is the root of the following characteristic equation:

$$D_2^2(1+\lambda)^4 + (2D_1D_2 - 4D_1^2 - D_2^2 + 6D_1D_2\cos\lambda\pi)(1+\lambda)^2 + D_1^2(1+\cos\lambda\pi)^2 + D_3^2\sin^2\lambda\pi = 0, \tag{1.10}$$

which is obtained by using a group approach [8]. Here, $D_1 = k(\varkappa_2 - 1) - (\varkappa_1 - 1)$; $D_2 = 4(1-k)$; $D_3 = k(\varkappa_2 + 1) - (\varkappa_1 + 1)$; $k = G_1/G_2$; $\varkappa_i = (3 - \nu_i)/(1 + \nu_i)$; and, the indices 1 and 2 correspond to the fiber and the matrix.

The roots of Eq. (1.10), lying in the region $-1 < \lambda < 0$, do not depend significantly on changes in the parameters and fall into the interval $-0.419 \le \lambda \le -0.394$ (with $\nu_1 = \nu_2 = 0.3$, $\lambda = -0.412$).

2. We shall now examine an analogous axisymmetrical problem. Assume that a fiber with a circular transverse cross section is pulled out of a matrix shaped like a semiinfinite cylinder ($a \le r \le b$, $0 \le z < \infty$), whose lateral surface ($r = b$) is fixed. The central line of the fiber is perpendicular to the plane $z = 0$ and coincides with the Oz axis. It is necessary to determine the law governing the distribution of the contact stresses between the fiber and the matrix, when an axial load with a resulting P_0 oriented along the fiber axis acts in the end section ($z = 0$) of the fiber.

In formulating spatial contact problems for bodies with elastic inclusions with a small transverse cross section, the model of a one-dimensional elastic rod together with the line contact model is not directly applicable [9]. For this reason, retaining for the fiber the model of an elastic rod, we assume that it is in contact with the matrix along the cylindrical surface.

In this formulation and in accordance with the separation of the stress–strain state of the matrix [10, 11], the solution (in the first approximation) reduces to integrating the equation for the fiber

$$d^2w_1/dz^2 = (P_0\delta(z) - \tau(z))/EF \tag{2.1}$$

and an approximate equation of equilibrium of the matrix

$$\partial^2 w/\partial r^2 + (1/r)\partial w/\partial r + \omega^2\partial^2 w/\partial z^2 = 0, \tag{2.2}$$

to which the following boundary conditions correspond:

$$w_z = 0 \ \ (z = 0), \ w = w_1 \, (r = a), \ w = 0 \, (r = b), \tag{2.3}$$

and all functions vanish at infinity. Here E and F are the modulus of elasticity of the material and the area of the transverse cross section of the fiber; $w(w_1)$ are the normal (along the z axis) displacements of the matrix (fiber); $\tau(z) = 2\pi a G w_r|_{r=a}$ is the contact stress per unit length of the fiber, which must be determined.

After applying the Fourier cosine transformation with respect to the z coordinate to the equations and boundary conditions, solving the ordinary differential equation obtained from (2.2), and calculating the inverse transforms, we find

$$\tau(z) = \frac{2P_0 g_1}{\pi}\int_0^\infty \frac{\cos zs^1}{sf(s) + g_1}\,ds, \tag{2.4}$$

where $g_1 = \frac{2\pi a G\omega}{EF} = \frac{2G\omega}{Ea}$; $f(s) = \frac{I_0(\omega bs)K_0(\omega as) - I_0(\omega as)K_0(\omega bs)}{I_0(\omega bs)K_1(\omega as) + I_1(\omega as)K_0(\omega bs)}$; $I_k(x)$, $K_k(x)$ $(k = 0, 1)$ are modified Bessel functions.

For large values of the parameter s, which corresponds to small values of the z coordinate, $f(s) \approx 1$, while the contact stress $\tau(z)$ is expressed as follows:

$$\tau(z) = \frac{2P_0 g_1}{\pi}\int_0^\infty \frac{\cos zs}{s + g_1}\,ds = -\frac{2P_0 g_1}{\pi}(\cos g_1 z \,\mathrm{ci}\, g_1 z + \sin g_1 z \,\mathrm{si}\, g_1 z). \tag{2.5}$$

The distribution of the contact stresses (2.5) corresponds to the solution of the problem of pulling a fiber out of a semiinfinite matrix; as in the two dimensional problem, we have a logarithmic singularity at $z = 0$.

For small s, which corresponds to large values of the z coordinate,

$$f(s) \approx [1 + \varkappa(s)]^{-1}\omega as \ln(b/a), \ \varkappa(s) = ((\omega as)^2/2)\ln(2/\gamma\omega bs),$$

γ is Euler's constant. For small s, it is possible to neglect the function $\varkappa(s)$ compared to unity, when

$$f(s) \approx \omega as \ln(b/a).$$

In this case, the stress (2.4) takes the form

$$\tau(z) = \frac{2P_0 g_{1*}^2}{\pi}\int_0^\infty \frac{\cos zs}{s^2 + g_{1*}^2}\,ds = P_0 g_{1*} e^{-g_{1*}z}, \ g_{1*}^2 = \frac{g_1}{\omega a \ln\frac{b}{a}} = \frac{2G}{Ea^2\ln\frac{b}{a}}. \tag{2.6}$$

Equations (1.6) and (2.6) are identical, but the latter does not correspond to the distribution of contact stresses in a discrete composite material.

Thus, in the spatial problem, the solution obtained with the simplified approach, when it is assumed that the fiber functions only under tension while the matrix functions only under shear, does not correspond to the asymptotic behavior of the exact solution. For this reason, such a simplified approach can be viewed as justified only in the two-dimensional problem and, in addition, its range of applicability is limited.

Analogous results were obtained in examining the axisymmetrical problem of pulling a fiber out of a matrix shaped like a finite cylinder ($a \le r \le b$, $0 \le z \le h$), whose lateral surface ($r = b$) is fixed.

As in the two-dimensional case, the form of the singular behavior of the contact stresses in the vicinity of the singular line $r = a$, $z = 0$, is established based on the complete equations of the theory of elasticity. In this case, the characteristic values λ of the singular solution turn out to be the same as in the two- and three-dimensional problems, if the boundary conditions at the boundary of the fiber are identical.

3. The solutions obtained above for the contact stresses, which are each valid in their own zone of the contact region, must transform smoothly into one another. The relation between the solutions of two neighboring zones can be established as follows. In one of the zones, the solution is known completely. In the solution of the neighboring zone, a coefficient is unknown. The requirement that the solutions and their derivatives coincide permits finding the unknown coefficient and the coordinate of the point at which these two solutions are joined.

As an example, we shall join the solutions (1.8), which includes solutions (1.7) and (1.9), with the singular solution near the fiber–matrix joint at a free boundary ($x = 0$), when the stressed state of the fiber in the given neighborhood is described by the equations of the two-dimensional stressed state. In this case,

$$\tau_1(\xi) = A_1\xi^\lambda,\ \tau_2(\xi) = (2P_0/\pi b)g_2\varphi(\xi),$$

where $\varphi(\xi) = \sum_{n=1}^{\infty} \frac{\cos(n\pi\beta^{-1}\xi)}{n\,\mathrm{th}(2n\pi\beta^{-1}) + 2g_2\beta\pi^{-1}}$; $\xi = \frac{x}{b}$; $\beta = \frac{h}{b}$; $g_2 = \frac{2Gb}{EF}$; $\omega = 2$. Requiring that the conditions $\tau_1 = \tau_2$, $\tau_1' = \tau_2'$ be satisfied, we obtain

$$A_1 = \frac{2P_0}{\pi b} A_1^*,\ A_1^* = g_2\xi_*^{-\lambda}\varphi(\xi_*).$$

Here ξ_* (the point at which the solutions are joined) is the root of the equation

$$\lambda\varphi(\xi) - \xi\varphi'(\xi) = 0.$$

In particular, with $\lambda = -0.4$ and $\beta = 1$, for the values $g_2 = 0.01$, 0.1, 0.3, 0.5, 0.8, and 1.0, we find the following values of ξ_* and A_1^*, respectively: $\xi_* = 0.026$, 0.024, 0.02, 0.019, 0.017, 0.015; $A_1^* = 0.006$, 0.056, 0.157, 0.248, 0.368, 0.441. For $\beta = 2$ and the same values of λ and g_2, we have $\xi_* = 0.053$, 0.045, 0.037, 0.029, 0.022, 0.021; $A_1^* = 0.008$, 0.072, 0.191, 0.291, 0.418, 0.493.

The solutions are joined in an analogous manner in the case of axisymmetrical problems.

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STABILITY OF PLASTIC ELONGATION OF A BIMETALLIC SHEET

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The extension of a sheet is limited by the magnitude of the critical strain at which local thinning of the material begins with the formation of a neck. We solve the problem of the stability of plastic extension of a bimetallic sheet under conditions of plane strain. The solution is constructed by using the theory of finite deformations of a rigid-plastic material.

1. We consider the plastic extension of a bimetallic sheet with a given law of variation of length. The loss of stability in this case can be represented as a process of continuous change of equilibrium shapes. Therefore the critical strain at which a neck is produced can be determined by the bifurcation method.

Since the loss of stability of deformation under consideration occurs during the plastic deformations developed, we neglect elastic deformations and assume a model of a rigid-plastic materials with isotropic hardening.

The flow curves of the materials of the layers of the bimetallic sheet $\sigma_e^{(1)} = \sigma_e^{(1)}(e_e)$ and $\sigma_e^{(2)} = \sigma_e^{(2)}(e_e)$ are assumed given. Here superscripts 1 and 2 denote quantities referring to the separate layers of the sheet; $\sigma_e = [(3/2)s_{ij}s_{ij}]^{1/2}$, stress intensity; $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$, components of the stress deviator; σ_{ij}, components of the stress tensor; $\sigma = (1/3)\sigma_{mn}\delta_{mn}$, hydrostatic pressure; and e_e, cumulative plastic deformation.

The problem is to determine the strain e_{e*} beyond which deformation occurs with the formation of a neck.

As the equations of state we take the equations of the deformation theory of plasticity, written for finite strains in the form

$$s_{ij} = \frac{2}{3}\frac{\sigma_e}{e_e} e_{ij}, \tag{1.1}$$

where the e_{ij} are the logarithmic strains; $e_e = [(2/3)e_{ij}e_{ij}]^{1/2}$ is the intensity of the logarithmic strains. Logarithmic strains are used in Eqs. (1.1), since for large deformations the condition of incompressibility of the material $e_{ij}\delta_{ij} = 0$ is compatible with Eqs. (1.1) only for logarithmic strains.

Bifurcation in a state A means that in addition to unperturbed deformation in state A, perturbed deformation in a state B infinitely close to it is possible.

We introduce a Cartesian coordinate system in state A in such a way that for the same particles the coordinates x_i' in state B and the coordinate x_i in state A are related by the equation

$$x_i' = x_i + u_i, \tag{1.2}$$

where u_i is an additional infinitesimal displacement of a particle. Then

$$dx_i' = (\delta_{ij} + u_{i,j})\, dx_j. \tag{1.3}$$

From now on subscripts after a comma denote differentiation with respect to the corresponding coordinate x_i.

Retaining only first order infinitesimals in the determinant of system (1.3), we obtain the incompressibility condition of the medium in the form

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